Disordered and deconfined: exotic quantum criticality with Dirac fermions

Joseph Maciejko
University of Alberta

Shoucheng Zhang Memorial Workshop
Stanford University
May 4, 2019
Atoms, molecules and quantum liquids are made of elementary particles at very high energies. But at low energies, they interact strongly with each other to form quasi-particles, which look very much like the elementary particles themselves! Over the past forty years, we have learned that the strong correlation of these matter degrees of freedom does not lead to ugliness and chaos, but rather to extraordinary beauty and simplicity.

S.-C. Zhang, “To see a world in a grain of sand”, hep-th/0210162
Dirac fermions in CMP

- high-$T_C$ superconductors
- superfluid $^3$He-B
- graphene
- topological insulators
Emergent Dirac fermions in CMP

Kitaev spin liquids

$\alpha$-RuCl$_3$?

kagomé antiferromagnets: Dirac spin liquids? (Hastings, PRB ‘00; Ran et al., PRL ‘07)

ZnCu$_3$(OH)$_6$Cl$_2$

(Kitaev, Ann. Phys. ‘06)
interactions = ?
Dirac point + interactions

- DOS vanishes at zero chemical potential: necessary existence of quantum critical phenomena (could be 1st order)
Dirac “Mott” quantum criticality

- SU(N) Hubbard model on honeycomb lattice: semimetal to Kekulé-VBS transition (continuous)

\[ H = -t \sum_{\langle ij \rangle} [c_{i\alpha}^+ c_{j\alpha} + \text{h.c.}] - \frac{J}{2N} \sum_{\langle ij \rangle} [c_{i\alpha}^+ c_{j\alpha} + \text{h.c.}]^2 \]

Li, Jiang, Jian, Yao, Nat. Comm. ’17
Zhou et al., PRB ‘16 (QMC)
Dirac “Mott” quantum criticality

- SU(N) Hubbard model on honeycomb lattice: semimetal to Kekulé-VBS transition (continuous)

\[ H = -t \sum_{\langle ij \rangle} [c_{i \alpha}^+ c_{j \alpha} + \text{h.c.}] - \frac{J}{2N} \sum_{\langle ij \rangle} [c_{i \alpha}^+ c_{j \alpha} + \text{h.c.}]^2 \]

Li, Jiang, Jian, Yao, Nat. Comm. ’17
Zhou et al., PRB ‘16 (QMC)
Dirac “Mott” quantum criticality

- SU(N) Hubbard model on honeycomb lattice: semimetal to Kekulé-VBS transition (continuous)

\[ H = -t \sum_{\langle ij \rangle} [c_{i\alpha}^+ c_{j\alpha} + \text{h.c.}] - \frac{J}{2N} \sum_{\langle ij \rangle} [c_{i\alpha}^+ c_{j\alpha} + \text{h.c.}]^2 \]

Li, Jiang, Jian, Yao, Nat. Comm. ’17
Zhou et al., PRB ‘16 (QMC)

universal properties?
Dirac “Mott” quantum criticality

- VBS order parameter \( \phi = |\phi|e^{i\theta} \sim \text{XY vector at long wavelengths (C}_3 \text{ anisotropy irrelevant at criticality)} \)
- Continuum effective theory = XY Gross-Neveu-Yukawa model

\[
\mathcal{L} = \sum_{a=1}^{N} \bar{\psi}_a \dot{\psi}_a + g|\phi| \sum_{a=1}^{N} \bar{\psi}_a e^{i\theta_5} \psi_a \\
+ |\partial_\mu \phi|^2 + r|\phi|^2 + u|\phi|^4
\]
Dirac “Mott” quantum criticality

- VBS order parameter $\phi = |\phi|e^{i\theta} \sim$ XY vector at long wavelengths ($C_3$ anisotropy irrelevant at criticality)
- Continuum effective theory = XY Gross-Neveu-Yukawa model

$$\mathcal{L} = \sum_{a=1}^{N} \bar{\psi}_a \partial^\mu \psi_a + g|\phi| \sum_{a=1}^{N} \bar{\psi}_a e^{i\theta\gamma^5} \psi_a$$

\[ + |\partial_\mu \phi|^2 + r|\phi|^2 + u|\phi|^4 \]

$\xi \sim (J - J_C)^{-\nu}$

- Different from pure 3D XY ($\nu_{3DXY} = 0.67$)

<table>
<thead>
<tr>
<th>N</th>
<th>Method</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Large-N (present)</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>QMC (present)</td>
<td>1.06(5)</td>
</tr>
<tr>
<td></td>
<td>$4 - \epsilon$, Two-loop$^{28}$</td>
<td>0.94</td>
</tr>
<tr>
<td>6</td>
<td>Large-N (present)</td>
<td>1.26</td>
</tr>
<tr>
<td></td>
<td>QMC (present)</td>
<td>1.07(4)</td>
</tr>
<tr>
<td></td>
<td>$4 - \epsilon$, Two-loop$^{28}$</td>
<td>0.96</td>
</tr>
<tr>
<td>8</td>
<td>Large-N (present)</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>QMC (present)</td>
<td>1.11(3)</td>
</tr>
<tr>
<td></td>
<td>$4 - \epsilon$, Two-loop$^{28}$</td>
<td>0.97</td>
</tr>
<tr>
<td>10</td>
<td>Large-N (present)</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td>QMC (present)</td>
<td>1.07(2)</td>
</tr>
<tr>
<td></td>
<td>$4 - \epsilon$, Two-loop$^{28}$</td>
<td>0.97</td>
</tr>
<tr>
<td>12</td>
<td>QMC (present)</td>
<td>1.06(3)</td>
</tr>
<tr>
<td></td>
<td>$4 - \epsilon$, Two-loop$^{28}$</td>
<td>0.98</td>
</tr>
</tbody>
</table>
Dirac “Mott” quantum criticality

• VBS order parameter $\phi = |\phi|e^{i\theta} \sim$ XY vector at long wavelengths ($C_3$ anisotropy irrelevant at criticality)

• Continuum effective theory = XY Gross-Neveu-Yukawa model

$$L = \sum_{a=1}^{N} \bar{\psi}_a \not{\partial} \psi_a + g |\phi| \sum_{a=1}^{N} \bar{\psi}_a e^{i\theta} \gamma^5 \psi_a$$

$$+ |\partial_\mu \phi|^2 + r |\phi|^2 + u |\phi|^4$$

$$\chi_{\text{OP}}(k) \sim \frac{1}{|k|^{2-\eta_\phi}}$$

• Large anomalous dimension: OP fluctuations decay into electron-hole pairs

<table>
<thead>
<tr>
<th>$N$</th>
<th>Method</th>
<th>$\eta_\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Large-N (present)</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>QMC (present)</td>
<td>0.71(3)</td>
</tr>
<tr>
<td></td>
<td>$4 - \epsilon$, Two-loop $^{28}$</td>
<td>0.67</td>
</tr>
<tr>
<td>6</td>
<td>Large-N (present)</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>QMC (present)</td>
<td>0.78(2)</td>
</tr>
<tr>
<td></td>
<td>$4 - \epsilon$, Two-loop $^{28}$</td>
<td>0.77</td>
</tr>
<tr>
<td>8</td>
<td>Large-N (present)</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>QMC (present)</td>
<td>0.80(4)</td>
</tr>
<tr>
<td></td>
<td>$4 - \epsilon$, Two-loop $^{28}$</td>
<td>0.82</td>
</tr>
<tr>
<td>10</td>
<td>Large-N (present)</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>QMC (present)</td>
<td>0.85(4)</td>
</tr>
<tr>
<td></td>
<td>$4 - \epsilon$, Two-loop $^{28}$</td>
<td>0.86</td>
</tr>
<tr>
<td>12</td>
<td>Large-N (present)</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>QMC (present)</td>
<td>0.87(4)</td>
</tr>
<tr>
<td></td>
<td>$4 - \epsilon$, Two-loop $^{28}$</td>
<td>0.88</td>
</tr>
</tbody>
</table>

$\eta_{3D_{\text{XY}}} = 0.04$
Twisted bilayer graphene?

- Effective “cluster” Hubbard model on honeycomb lattice

\[ H_U = U \sum_{\bigtriangleup} (Q_{\bigtriangleup} - Q_0)^2 \]

Yuan & Fu, PRB ’18
Po, Zou, Vishwanath, Senthil, PRX ’18
Koshin et al., PRX ’18
Kang & Vafek, PRX ’18

Xu, Law, Lee, PRB ’18

Is magic-angle twisted bilayer graphene near a quantum critical point?

Yuan Da Liao,\(^1,2\) Zi Yang Meng,\(^1,3,4\) and Xiao Yan Xu\(^5,6\)

arXiv:1901.11434
Twisted bilayer graphene?

- Effective “cluster” Hubbard model on honeycomb lattice

\[ H_U = U \sum_{\square} (Q_{\square} - Q_0)^2 \]

Yuan & Fu, PRB ’18
Po, Zou, Vishwanath, Senthil, PRX ’18
Koshin et al., PRX ’18
Kang & Vafek, PRX ’18

Yuan Da Liao,1,2 Zi Yang Meng,1,3,4 and Xiao Yan Xu5,+

arXiv:1901.11434
Dirac superconducting quantum criticality

- \textbf{N=1: single Dirac cone on surface of 3D topological insulator}

\[
\xi \sim (U - U_c)^{-\nu}
\]

\[
\chi_{\text{OP}}(k) \sim \frac{1}{|k|^{2-\eta_{\phi}}}
\]

Li, Vaezi, Mendl, Yao, Sci. Adv. ’18
(QMC on long-range hopping model)

<table>
<thead>
<tr>
<th></th>
<th>(\nu)</th>
<th>(\eta_{\phi})</th>
</tr>
</thead>
<tbody>
<tr>
<td>QMC</td>
<td>0.87</td>
<td>0.34</td>
</tr>
<tr>
<td>(4-\epsilon)</td>
<td>0.88</td>
<td>1/3</td>
</tr>
<tr>
<td>conformal bootstrap</td>
<td>0.917</td>
<td>1/3</td>
</tr>
</tbody>
</table>
Dirac superconducting quantum criticality

- $N=1$: single Dirac cone on surface of 3D topological insulator

\[
\xi \sim (U - U_c)^{-\nu}
\]
\[
\chi_{OP}(k) \sim \frac{1}{|k|^{2-\eta_\phi}}
\]

Li, Vaezi, Mendl, Yao, Sci. Adv. ’18
(QMC on long-range hopping model)

<table>
<thead>
<tr>
<th>Method</th>
<th>$\nu$</th>
<th>$\eta_{\phi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>QMC</td>
<td>0.87</td>
<td>0.34</td>
</tr>
<tr>
<td>4-\epsilon</td>
<td>0.88</td>
<td>1/3</td>
</tr>
<tr>
<td>conformal bootstrap</td>
<td>0.917</td>
<td>1/3</td>
</tr>
</tbody>
</table>
Dirac superconducting quantum criticality

- $N=1$: single Dirac cone on surface of 3D topological insulator

Li, Vaezi, Mendl, Yao, Sci. Adv. ’18
(QMC on long-range hopping model)

$$A(k, \omega) \sim \frac{1}{(\omega^2 - v_F^2 k^2)^{(1-\eta_\psi)/2}}$$

<table>
<thead>
<tr>
<th>Method</th>
<th>$\nu$</th>
<th>$\eta_\phi$</th>
<th>$\eta_\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>QMC</td>
<td>0.87</td>
<td>0.34</td>
<td>0.37</td>
</tr>
<tr>
<td>4-$\epsilon$</td>
<td>0.88</td>
<td>1/3</td>
<td>1/3</td>
</tr>
<tr>
<td>conformal bootstrap</td>
<td>0.917</td>
<td>1/3</td>
<td>1/3</td>
</tr>
</tbody>
</table>

- Non-Fermi liquid behavior
Dirac superconducting quantum criticality

- N=1: single Dirac cone on surface of 3D topological insulator

Li, Vaezi, Mendl, Yao, Sci. Adv. '18
(QMC on long-range hopping model)

<table>
<thead>
<tr>
<th></th>
<th>( \nu )</th>
<th>( \eta_\phi )</th>
<th>( \eta_\psi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>QMC</td>
<td>0.87</td>
<td>0.34</td>
<td>0.37</td>
</tr>
<tr>
<td>4-( \varepsilon )</td>
<td>0.88</td>
<td>1/3</td>
<td>1/3</td>
</tr>
<tr>
<td>conformal bootstrap</td>
<td>0.917</td>
<td>1/3</td>
<td>1/3</td>
</tr>
</tbody>
</table>

\[ A(k, \omega) \sim \frac{1}{(\omega^2 - v_F^2 k^2)^{(1-\eta_\psi)/2}} \]
Dirac fermion quantum criticality

• How is QC behavior modified by disorder?
Dirac fermion quantum criticality

- How is QC behavior modified by disorder?
- How is QC behavior modified by coupling to emergent gauge fields?
Outline

- Disordered fermionic quantum critical points
  
  H. Yerzhakov and JM, PRB 98, 195142 (2018)

- Transitions in Dirac spin liquids

  R. Boyack, A. Rayyan, JM, arXiv:1812.02720
  N. Zerf, R. Boyack, P. Marquard, J. A. Gracey, JM, in preparation
Disordered fermionic QCPs

PRB 98, 035137 (2018)

H. Yerzhakov
(Alberta)
Phase transitions in disordered systems

- Focus on Dirac semimetal-superconductor transition with N two-component fermions (N = 1: TI surface, N = 4: graphene)
Phase transitions in disordered systems

- Focus on Dirac semimetal-superconductor transition with N two-component fermions (N = 1: TI surface, N = 4: graphene)

- Add quenched disorder

Does the transition still exist?
Phase transitions in disordered systems

- Focus on Dirac semimetal-superconductor transition with \( N \) two-component fermions (\( N = 1 \): TI surface, \( N = 4 \): graphene)

- Add quenched disorder

Diagram:

1. Does the transition still exist? **no**
2. Is it still continuous? **yes**
Phase transitions in disordered systems

- Focus on Dirac semimetal-superconductor transition with \( N \) two-component fermions (\( N = 1 \): TI surface, \( N = 4 \): graphene)

- Add quenched disorder

\begin{itemize}
  \item does the transition still exist?
    \begin{itemize}
      \item yes
    \end{itemize}

  \item is it still continuous?
    \begin{itemize}
      \item yes
      \item no
    \end{itemize}

  \item is it a new universality class?
    \begin{itemize}
      \item yes
      \item no
    \end{itemize}
\end{itemize}
Phase transitions in disordered systems

- Focus on Dirac semimetal-superconductor transition with $N$ two-component fermions ($N = 1$: TI surface, $N = 4$: graphene)

- Add quenched disorder

- Does the transition still exist?
  - Yes

- Is it still continuous?
  - Yes

- Is it a new universality class?
  - Yes
A tale of two disorders

• **Random-field** (symmetry-breaking) disorder: for continuous symmetries, destroys long-range order in $d \leq 4$ dimensions (Imry-Ma)

• **Random-coupling** (symmetry-preserving) disorder: locally superconducting islands

$$\Delta(x) \propto (g(x) - g_c)^{z\nu}$$
A tale of two disorders

- **Random-field** (symmetry-breaking) disorder: for continuous symmetries, destroys long-range order in $d \leq 4$ dimensions (Imry-Ma)

- For superconducting transition, no random field: random potentials are particle-number conserving!

- **Random-coupling** (symmetry-preserving) disorder: XY GNY model with random boson mass

\[
\mathcal{L} = \left| \partial_\mu \phi \right|^2 + (r_0 + \delta r(\mathbf{x}))|\phi|^2 + u|\phi|^4
\]

\[
= \frac{1}{g_c} - \frac{1}{g(\mathbf{x})} + \text{fermions}
\]
The Harris criterion: XY GNY model

- $(2+1)D$ QCP stable against disorder provided $\nu_{\text{clean}} > 1$ (Harris)
The Harris criterion: XY GNY model

- (2+1)D QCP stable against disorder provided $\nu_{\text{clean}} > 1$ (Harris)

<table>
<thead>
<tr>
<th>N=1</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>QMC</td>
<td>0.87</td>
</tr>
<tr>
<td>4-$\epsilon$</td>
<td>0.88</td>
</tr>
<tr>
<td>conformal bootstrap</td>
<td>0.917</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>N</th>
<th>Method</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Large-$N$ (present)</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>QMC (present)</td>
<td>1.06(5)</td>
</tr>
<tr>
<td></td>
<td>4-$\epsilon$, Two-loop$^{28}$</td>
<td>0.94</td>
</tr>
<tr>
<td>6</td>
<td>Large-$N$ (present)</td>
<td>1.26</td>
</tr>
<tr>
<td></td>
<td>QMC (present)</td>
<td>1.07(4)</td>
</tr>
<tr>
<td></td>
<td>4-$\epsilon$, Two-loop$^{28}$</td>
<td>0.96</td>
</tr>
<tr>
<td>8</td>
<td>Large-$N$ (present)</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>QMC (present)</td>
<td>1.11(3)</td>
</tr>
<tr>
<td></td>
<td>4-$\epsilon$, Two-loop$^{28}$</td>
<td>0.97</td>
</tr>
<tr>
<td>10</td>
<td>Large-$N$ (present)</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td>QMC (present)</td>
<td>1.07(2)</td>
</tr>
<tr>
<td></td>
<td>4-$\epsilon$, Two-loop$^{28}$</td>
<td>0.97</td>
</tr>
<tr>
<td>12</td>
<td>Large-$N$ (present)</td>
<td>1.22</td>
</tr>
<tr>
<td></td>
<td>QMC (present)</td>
<td>1.06(3)</td>
</tr>
<tr>
<td></td>
<td>4-$\epsilon$, Two-loop$^{28}$</td>
<td>0.98</td>
</tr>
</tbody>
</table>

stable according to QMC
The Harris criterion: XY GNY model

- (2+1)D QCP stable against disorder provided $\nu_{\text{clean}} > 1$ (Harris)

<table>
<thead>
<tr>
<th>$N=1$ Method</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>QMC</td>
<td>0.87</td>
</tr>
<tr>
<td>$4 - \epsilon$, conformal bootstrap</td>
<td>0.88</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$N$ Method</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 Large-N (present)</td>
<td>1.25</td>
</tr>
<tr>
<td>QMC (present)</td>
<td>1.06(5)</td>
</tr>
<tr>
<td>$4 - \epsilon$, Two-loop $^{28}$</td>
<td>0.94</td>
</tr>
<tr>
<td>6 Large-N (present)</td>
<td>1.26</td>
</tr>
<tr>
<td>QMC (present)</td>
<td>1.07(4)</td>
</tr>
<tr>
<td>$4 - \epsilon$, Two-loop $^{28}$</td>
<td>0.96</td>
</tr>
<tr>
<td>8 Large-N (present)</td>
<td>1.25</td>
</tr>
<tr>
<td>QMC (present)</td>
<td>1.11(3)</td>
</tr>
<tr>
<td>$4 - \epsilon$, Two-loop $^{28}$</td>
<td>0.97</td>
</tr>
<tr>
<td>10 Large-N (present)</td>
<td>1.23</td>
</tr>
<tr>
<td>QMC (present)</td>
<td>1.07(2)</td>
</tr>
<tr>
<td>$4 - \epsilon$, Two-loop $^{28}$</td>
<td>0.97</td>
</tr>
<tr>
<td>12 Large-N (present)</td>
<td>1.22</td>
</tr>
<tr>
<td>QMC (present)</td>
<td>1.06(3)</td>
</tr>
<tr>
<td>$4 - \epsilon$, Two-loop $^{28}$</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Unstable does the transition still exist? Stable according to QMC
Double epsilon expansion

- Replica trick + double epsilon expansion RG: \( d = 4 - \epsilon \) space and \( \epsilon_T \) time dimensions (Dorogovtsev, PLA '80; Boyanovsky, Cardy, PRB '82)
Double epsilon expansion

- Replica trick + double epsilon expansion RG: $d = 4 - \epsilon$ space and $\epsilon_T$ time dimensions (Dorogovtsev, PLA ’80; Boyanovsky, Cardy, PRB ’82)

- Purely bosonic limit: reduces to random-$T_c$ O(2) vector model, good agreement with QMC studies of boson superfluid-Mott glass transition (Vojta et al., PRB ‘16; Crewse, Lerch, Vojta, PRB ‘18)

<table>
<thead>
<tr>
<th>$d$</th>
<th>$\nu$</th>
<th>$\tilde{z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2, QMC</td>
<td>1.16(5)</td>
<td>1.52(3)</td>
</tr>
<tr>
<td>2, 1-loop RG</td>
<td>1.125</td>
<td>1.75</td>
</tr>
<tr>
<td>3, QMC</td>
<td>0.90(5)</td>
<td>1.67(6)</td>
</tr>
<tr>
<td>3, 1-loop RG</td>
<td>0.9375</td>
<td>1.625</td>
</tr>
</tbody>
</table>

disorder-induced vertex
RG flow: \( N = 1 \)

\[ |\phi|^4 \text{ coupling} \]

- 3D XY
- Gaussian

- Disorder (marginally) relevant (Harris), runaway flow

- Clean QCP

- \( T^\ast \sim \Lambda e^{-1/\Delta_0} \)

- SM
- SC

\( g_{\text{BCS}} \)
RG flow: $N > 1$

$|\phi|^4$ coupling

3D XY

Gaussian

disorder

Yukawa coupling

clean QCP

disorder perturbatively irrelevant (Harris)
RG flow: $N > 1$

$|\phi|^4$ coupling

3D XY

Gaussian

disorder

Yukawa coupling

disorder perturbatively irrelevant (Harris)

is it a new universality class?

no
RG flow: $N > 1$

$|\phi|^4$ coupling

3D XY

Gaussian

Yukawa coupling

disorder

new finite-disorder, fermionic (multi)critical points
Schematic phase diagram

(a) $r$ N = 2, 3

SM

DFP 2

DFP 1

CFP

SC

$\Delta$ (disorder)

(b) $r$ N = 4

SM

CH

DFP

SC

$\Delta$

(c) $r$ N $\geq$ 5

SM

DFP 1

CFP

DFP 2

SC

$\Delta$

Disordered fixed points are “Harris stable” too (Chayes inequality)

$\nu^{-1} < d/2$
Schematic phase diagram

(a) $r$ with $N = 2, 3$

- SM
- DFP 2
- CFP
- DFP 1
- SC

(b) $r$ with $N = 4$

- SM
- DFP
- CFP
- SC

(c) $r$ with $N \geq 5$

- SM
- DFP 1
- CFP
- DFP 2
- SC

$\Delta$ (disorder)

Graph showing noninteger dynamic critical exponent $z > 1$
Stable-focus fixed points

• For all $N \geq 7$, DFP 2 is of stable-focus type (spiral): irrelevant RG eigenvalues are complex

• Oscillatory corrections to scaling (Khmelnitskii, PLA '78)

$$\chi \sim |r|^{-\gamma} \left[ 1 + C \left| \frac{r}{r_0} \right|^{\nu \omega'} \right] \cos \left( \nu \omega'' \ln \left| \frac{r}{r_0} \right| + \phi \right) + \ldots$$
Transitions in Dirac spin liquids

PRB 98, 035137 (2018)
PRB 98, 165125 (2018)
arXiv:1812.02720

R. Boyack (Alberta)
C.-H. Lin (UMN)
N. Zerf (HU Berlin)
A. Rayyan (Alberta)
P. Marquard (DESY)
Spin liquids

System of local moments with no magnetic order down to $T=0$ (Anderson, Mat. Res. Bull. ‘73)
Gapped spin liquids: proofs of principle

System of local moments with no magnetic order down to $T=0$ (Anderson, Mat. Res. Bull. ‘73)

Kitaev toric code

Kitaev honeycomb model + weak $B_{[111]}$ field
Gapped spin liquids: experiment

System of local moments with no magnetic order down to T=0 (Anderson, Mat. Res. Bull. ‘73)

α-RuCl₃

Kasahara et al., Nature ‘18

Kitaev honeycomb model + weak B[111] field
Gapless spin liquids: experiment

Yamashita et al., Nature ’08

Yamashita et al., Nat. Comm. ’11

Han et al., Nature ’12
Gapless spin liquids: proofs of principle?

- Gapless
- U(1) spinon
  - Fermi surface
- U(1) Dirac/algebraic
Gapless spin liquids: proofs of principle?

- Gapless

  \[ \Downarrow \]

  \begin{align*}
  &\text{U(1) spinon} \\
  &\text{Fermi surface} \\
  &? \\
  &\Downarrow \\
  \end{align*}

- \text{U(1) Dirac/algebraic}

  \[ \Downarrow \]

  \begin{align*}
  &? \\
  \end{align*}
U(1) Dirac spin liquid: proof of principle

\[ H = \frac{1}{2}J N_f \sum_{\langle i,j \rangle} \frac{1}{4} \hat{L}_{ij}^2 - t \sum_{\langle i,j \rangle, \alpha} \left( \hat{c}_{i \alpha}^\dagger e^{i \hat{\phi}_{ij}} \hat{c}_{j \alpha} + \text{h.c.} \right) + \frac{1}{2} K N_f \sum \cos \left( \text{curl} \hat{\theta} \right) \]

Xu et al., PRX '19
(QMC)
U(1) Dirac spin liquid: proof of principle

\[ H = \frac{1}{2} J N_f \sum_{\langle i,j \rangle} \frac{1}{4} \hat{L}_{ij}^2 - t \sum_{\langle i,j \rangle \alpha} \left( \hat{c}_{i \alpha}^\dagger e^{i \hat{\theta}_{ij}} \hat{c}_{j \alpha} + \text{h.c.} \right) \]

\[ + \frac{1}{2} K N_f \sum \cos \left( \text{curl} \hat{\theta} \right) \]

\[ c_i \text{ or } c_i^\dagger \]

Affleck-Marston \( \pi \)-flux phase

Xu et al., PRX '19 (QMC)

Universal power-law correlations (Rantner, Wen, Hermele, Senthil, Fisher…)

Real space correlation

\[ |r_i - r_j| \]

\[ N_f = 2 \]
U(1) Dirac spin liquid: proof of principle

\[ H = \frac{1}{2} J N_f \sum_{\langle i,j \rangle} \frac{1}{4} \hat{L}_{ij}^2 - t \sum_{\langle i,j \rangle \alpha} \left( \hat{c}_{i\alpha}^\dagger e^{i\hat{\theta}_{ij}} \hat{c}_{j\alpha} + \text{h.c.} \right) + \frac{1}{2} K N_f \sum_{\square} \cos \left( \text{curl} \hat{\theta} \right) \]

QCP? \quad J

\[ N_f = 2 \]

Xu et al., PRX '19 (QMC)

Universal power-law correlations
(Rantner, Wen, Hermele, Senthil, Fisher…)

Affleck-Marston $\pi$-flux phase

Graphical representation of the model with $N_f = 2$ showing different phases such as AFM and VBS as a function of $N_f$.
U(1) Dirac

- Continuum effective theory:

\[ \mathcal{L} = \sum_{i=1}^{N_f} \bar{\Psi}_i \not{D} \Psi_i + \frac{1}{4} F_{\mu \nu}^2 \]

QED$_3$
U(1) Dirac-to-Néel transition

- Continuum effective theory: Heisenberg QED$_3$-GNY model

\[ \mathcal{L} = \sum_{i=1}^{N_f} \overline{\Psi}_i \slashed{D} \Psi_i + \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 + \lambda^2 (\phi^2)^2 + g \phi \cdot \sum_{i=1}^{N_f} \overline{\Psi}_i \frac{\sigma}{2} \Psi_i \]

QCP? --- J ---

UID

AFM
4-\(\varepsilon\) expansion

\[(\phi^2)^2 \text{ coupling}\]

3D Heisenberg

Gaussian
4-ε expansion

\[(\phi^2)^2\] coupling

3D Heisenberg

Gaussian

Heisenberg GNY

Yukawa coupling
4-ε expansion

- Stable fixed point (4-loop order) for all $N_f$: continuous transition, agrees with QMC

\[
(\phi^2)^2 \text{ coupling}
\]

- 3D Heisenberg
- Gaussian
- Yukawa coupling
- Heisenberg GNY
- Heisenberg QED$_3$-GNY

Ghaemi & Senthil, PRB ‘06
Zerf, Boyack, Marquard, Gracey, JM, in preparation
Critical exponents

- \( \varepsilon \) and \( 1/N_f \) expansions, compare to QMC in future

\[
\langle S_r \cdot S'_r \rangle \sim \frac{1}{|r - r'|^{1+\eta_\phi}}
\]

\[
\langle \mathcal{O}_{CDW}(r) \mathcal{O}_{CDW}(r') \rangle \sim \frac{1}{|r - r'|^{2\Delta}}
\]

\[
\langle \mathcal{O}_{VBS}(r) \cdot \mathcal{O}_{VBS}(r') \rangle \sim \frac{1}{|r - r'|^{2\Delta}}
\]

Zerf, Boyack, Marquard, Gracey, JM, in preparation
Proximate phases

Néel

Heisenberg
QED$_3$-GNY

Proximate phases

Néel

Heisenberg
QED$_3$-GNY

Proximate phases

Néel

Heisenberg
QED$_3$-GNY
Proximate phases

Heisenberg QED$_3$-GNY

Néel

VBS
Proximate phases

\[ \mathcal{L} = \sum_{i=1}^{N} \overline{\psi}_i \slashed{D} \psi_i + g \phi \sum_{i=1}^{N} \overline{\psi}_i \psi_i + \ldots \]

- Néel
- Heisenberg QED$_3$-GNY
- XY QED$_3$-GNY
- Ising QED$_3$-GNY
- VBS
- Chiral spin liquid

Janssen and He, PRB '17
Ihrig et al., PRB '18
Zerf, Marquard, Boyack, JM, PRB '18
Proximate phases

\[ L = |D_\mu \bar{z}|^2 + \lambda (\bar{z}^\dagger z)^2 + \ldots \]

Heisenberg
QED\textsubscript{3}-GNY

\[ \mathcal{L} = \sum_{i=1}^{N} \bar{\psi}_i \not D \psi_i + g \phi \sum_{i=1}^{N} \bar{\psi}_i \psi_i + \ldots \]

Ising
QED\textsubscript{3}-GNY

QED\textsubscript{3}-GNY

deconfined
QCP

XY QED\textsubscript{3}-GNY

duality?

Néel

Senthil et al., Science ‘04

\[ \mathcal{L} = \sum_{i=1}^{N} \bar{\psi}_i \not D \psi_i + g \phi \sum_{i=1}^{N} \bar{\psi}_i \psi_i + \ldots \]

VBS

chiral spin liquid

Janssen and He, PRB ’17

Ihrig et al., PRB ’18

Zerf, Marquard, Boyack, JM, PRB ‘18
Deconfined Quantum Critical Points: Symmetries and Dualities

Chong Wang,1,2 Adam Nahum,3,4 Max A. Metlitski,3,5,2 Cenke Xu,6,2 and T. Senthil3

Néel-VBS deconfined QCP

$z^\dagger \sigma_z z$ (z-component of Néel OP)

Ising QED$_3$-GNY

$\phi$
Deconfined Quantum Critical Points: Symmetries and Dualities

Chong Wang,¹ ² Adam Nahum,³ ⁴ Max A. Metlitski,³ ⁵ ² Cenke Xu,⁶ ² and T. Senthil³

\[ \eta_{\text{Néel}} = \eta_{\phi} \]
Deconfined Quantum Critical Points: Symmetries and Dualities

Chong Wang, 1,2 Adam Nahum,3,4 Max A. Metlitski,3,5,2 Cenke Xu,6,2 and T. Senthil3

Néel-VBS deconfined QCP

Ising QED3-GNY

\[ z^\dagger \sigma z \]

(z-component of Néel OP)

\[ \eta_{\text{Néel}} = \eta_{\phi} \]

0.25 − 0.35

Sandvik, PRL ‘07
Melko & Kaul, PRL ‘08
Nahum et al., PRX ‘15
Deconfined Quantum Critical Points: Symmetries and Dualities

Chong Wang,1,2 Adam Nahum,3,4 Max A. Metlitski,3,5,2 Cenke Xu,6,2 and T. Senthil3

Néel-VBS deconfined QCP

Ising QED3-GNY

\[ \eta_{\text{Néel}} = \eta_{\phi} = 1 - \frac{96}{\pi^2 N} + \mathcal{O}(1/N^2) \]

\[ \{ 0.17, \text{ Padé} \}

\[ \{ 0.30, \text{ Padé-Borel} \]
Deconfined Quantum Critical Points: Symmetries and Dualities

Chong Wang, Adam Nahum, Max A. Metlitski, Cenke Xu, and T. Senthil

Néel-VBS deconfined QCP

relevant operator

Ising QED\(_3\)-GNY

\[ \bar{\psi} \sigma_z \psi \]
Deconfined Quantum Critical Points: Symmetries and Dualities

Chong Wang,1,2 Adam Nahum,3,4 Max A. Metlitski,3,5,2 Cenke Xu,6,2 and T. Senthil3

\[ 3 - \nu_{\text{Néel-VBS}}^{-1} = \Delta \bar{\psi} \sigma_z \psi \]
Deconfined Quantum Critical Points: Symmetries and Dualities

Chong Wang, Adam Nahum, Max A. Metlitski, Cenke Xu, and T. Senthil

\[ \text{Néel-VBS deconfined QCP} \quad \leftrightarrow \quad \text{Ising QED}_3\text{-GNY} \]

relevant operator

\[ 3 - \nu_{\text{Néel-VBS}}^{-1} = \Delta \bar{\psi} \sigma_z \psi \]

1.0 – 1.7

Sandvik, PRL ‘07
Melko & Kaul, PRL ‘08
Nahum et al., PRX ‘15
Néel-VBS deconfined QCP

Ising QED\(_3\)-GNY

relevant operator

\[
3 - \nu_{\text{Néel-VBS}}^{-1} = \Delta \overline{\psi} \sigma_z \psi = 2 - \frac{16}{\pi^2 N} + \mathcal{O}(1/N^2)
\]

\[
= \begin{cases} 
1.4, & \text{Padé} \\
1.5, & \text{Padé-Borel}
\end{cases}
\]

Boyack, Rayyan, JM, arXiv:1812.02720

Sandvik, PRL '07
Melko & Kaul, PRL '08
Nahum et al., PRX '15
Summary

• Dirac fermions…
  ⊙ … couple strongly to bosonic order parameter fluctuations: new universality classes of QC behavior
  ⊙ + quenched disorder: novel finite-disorder fermionic QCPs
  ⊙ + emergent gauge fields: novel QCPs describing transitions between U(1) Dirac SL and (1) ordered states, (2) gapped spin liquids ($Z_2$ or chiral)

• 1/N expansion supports Néel-VBS deconfined QCP / Ising QED$_3$-GNY duality conjecture
Summary

- Dirac fermions…
  - … couple strongly to bosonic order parameter fluctuations: new universality classes of QC behavior
  - + quenched disorder: novel finite-disorder fermionic QCPs
  - + emergent gauge fields: novel QCPs describing transitions between U(1) Dirac SL and (1) ordered states, (2) gapped spin liquids (Z₂ or chiral)
- 1/N expansion supports Néel-VBS deconfined QCP / Ising QED₃-GNY duality conjecture

Thank you!
Ongoing/future work

• Analytics:
  ᵇ Effect of long-range correlated disorder on Dirac QCPs?
  ᵇ Critical exponents of U(1) Dirac-VBS transition?
  ᵇ Effect of disorder on spin liquid transitions?

• Numerics:
  ᵇ Effect of disorder in “Dirac” (honeycomb lattice, $\pi$-flux, long-range hopping) Hubbard models?
  ᵇ Critical exponents of U(1) Dirac to Néel or VBS transitions?
  ᵇ Induce U(1) Dirac to gapped SL transitions?
Dirac “Mott” quantum criticality

- Repulsive spin-1/2 Hubbard model on honeycomb lattice: AF transition

Otsuka, Yunoki, Sorella, PRX ‘16

suppression of double occupancy at large U
Dirac “Mott” quantum criticality

- Repulsive spin-1/2 Hubbard model on honeycomb lattice: AF transition

universal QC properties?

suppression of double occupancy at large U
Dirac “Mott” quantum criticality

- Distinct from 3D Heisenberg universality class ($\nu_{3D\text{Heis}} = 0.7$)

\[ \xi \sim (U - U_c)^{-\nu} \]

\[ \nu = 1.02(1) \]
Dirac “Mott” quantum criticality

- Large anomalous dimension: OP fluctuations decay into electron-hole pairs

\[ \chi_{AF}(k) \sim \frac{1}{|k|^{2-\eta_\phi}} \]

\[ \eta_\phi = 0.70(15) \]

Parisen Toldin et al., PRB ‘15

\[ \eta_{\phi}^{3D \ Heis} = 0.04 \]
Dirac “Mott” quantum criticality

- Non-Fermi liquid behavior

\[ A(k, \omega) \sim \frac{1}{(\omega^2 - v_F^2 k^2)^{1-\eta_\psi}/2} \]

\[ \eta_\psi = 0.20(2) \]
# The Harris criterion

<table>
<thead>
<tr>
<th>\langle g_c(1) \rangle</th>
<th>\langle g_c(2) \rangle</th>
<th>\langle g_c(3) \rangle</th>
<th>\langle g_c(4) \rangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Variation in critical coupling between correlation volumes suppressed by $1/\sqrt{\# \text{ random variables in that volume}}$

\[
\Delta g_c \propto \frac{1}{\sqrt{\xi^d}}
\]

- Clean QCP remains well defined if

\[
\Delta g_c \ll g - g_c
\]

as $g \to g_c$
The Harris criterion

<table>
<thead>
<tr>
<th>$\langle g_c(1) \rangle$</th>
<th>$\langle g_c(2) \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle g_c(3) \rangle$</td>
<td>$\langle g_c(4) \rangle$</td>
</tr>
</tbody>
</table>

- Variation in critical coupling between correlation volumes suppressed by $1/\sqrt{\#}$ random variables in that volume

$$\Delta g_c \propto \frac{1}{\sqrt{\xi^d}}$$

- Clean QCP remains well defined if

$$\Delta g_c \ll g - g_c \sim \xi^{-1}/\nu_{\text{clean}}$$

as $g \to g_c \Leftrightarrow \xi \to \infty$

- Clean QCP stable against disorder provided $\nu_{\text{clean}}^{-1} < d/2$
RG flow: $N = 2$

in $h^2 = h^{-2}$ plane

disorder

$\Delta/\epsilon_T$

$\lambda^2/\epsilon_T$

$|\phi|^4$ coupling

DFP 1

DFP 2

CFP

separatrix
RG flow: $N = 2$ (N = 3 qualitatively similar, but DFP 1 & 2 become closer)

in $h^2 = h^{-2}$ plane

Disorder

$\Delta/\epsilon_T$

$\lambda^2/\epsilon_T$

$|\phi|^4$ coupling

separatrix

DFP 1

DFP 2

CFP

$h^2 = h^{-2}$ plane
RG flow: N = 4

DFP 1 & 2 merge into a single DFP: marginal scaling (Kaplan et al., PRD '09)
RG flow: $N = 5$

DFP 1 & 2 reappear, exchange stability properties

Disorder

$\lambda^2/\epsilon$ 

$|\phi|^4$ coupling

DFP 1

DFP 2

CFP
RG flow: N = 6  
(qualitatively similar to N = 5)
RG flow: \( N = 7 \)
RG flow: $N = 8$
RG flow: $N = 10$
RG flow: $N = 20$