Persistent Spin Helix Collaboration (2005-2010)

Chris Weber

Jake Koralek
Diffusion of Nonequilibrium Quasi-Particles in a Cuprate Superconductor

N. Gedik, J. Orenstein, Ruixing Liang, D. A. Bonn, W. N. Hardy

Phase coherent transient grating spectroscopy

Diffusion of particle density

Decay rate $\propto Dq^2$
Spin Gratings and the Measurement of Electron Drift Mobility in Multiple Quantum Well Semiconductors

A. R. Cameron, P. Riblet, and A. Miller
Select phase of $\delta r$ by simply rotating coverslip

- $\delta r$ signal
- local oscillator

Probed by diffraction and coherent mixing of two probes

Generation and detection of spin polarization wave
Naïve view $\delta$-doped GaAs quantum well

Proceeded, blissfully unaware of spin-orbit coupling

$\delta$-doped GaAs quantum well

Parabolic conduction band

Circular, spin-degenerate Fermi surface
Observation of spin Coulomb drag in a two-dimensional electron gas

C. P. Weber¹, N. Gedik¹, J. E. Moore¹, J. Orenstein¹, J. Stephens³ & D. D. Awschalom³

Hydrodynamic regime $\tau_{ee} \ll \tau_{mom}$
A cool idea – Datta-Das spin transistor

Problem: required 1-dimensional ballistic propagation

Electric field of gate tunes Rashba SO field
Solution: tune the two SO terms to be equal!

Rashba

\[ H_R = \alpha (k_y \sigma_x - k_x \sigma_y) \]

Tunable with \( E_{\text{ext}} \)

Dresselhaus

\[ H_D = \beta (k_x \sigma_x - k_y \sigma_y) \]

Tunable with \( d \)
What happens when Rashba = Dresselhaus
Net precession depends only on $\Delta x$.

For the same $x$ distance traveled, a spin precesses by exactly the same angle.
Datta-Das spin transistor survives scattering and 2-Dimensions

Electric field of gate tunes Rashba SO field
Think about what this means for a spin helix with the right pitch
Spin-orbit Fermi contours

Rashba only
Concentric Fermi surfaces with opposite chirality of spin texture

Rashba > Dresselhaus
Complicated Fermi contours

Rashba = Dresselhaus
Two identical but shifted FC’s
The Exact SU(2) Symmetry

- Finite wavevector spin components

\[ S_Q^- = \sum_{\vec{k}} c_{\vec{k}\downarrow}^+ c_{\vec{k} + \vec{Q}\uparrow}, \quad S_Q^+ = \sum_{\vec{k}} c_{\vec{k} + \vec{Q}\uparrow}^+ c_{\vec{k}\downarrow}, \quad S_Q^z = \sum_{\vec{k}} c_{\vec{k}\uparrow}^+ c_{\vec{k}\uparrow} - c_{\vec{k}\downarrow}^+ c_{\vec{k}\downarrow} \]

\[ \begin{bmatrix} S_0^z, S_Q^\pm \end{bmatrix} = \pm 2S_Q^\pm, \quad \begin{bmatrix} S_Q^+, S_Q^- \end{bmatrix} = S_0^z \]

- Shifting property essential

\[ \begin{bmatrix} H_{\text{ReD}}, c_{\vec{k} + \vec{Q}\uparrow}^+ c_{\vec{k}\downarrow} \end{bmatrix} = \left( \epsilon_{\uparrow}(\vec{k} + \vec{Q}) - \epsilon_{\downarrow}(\vec{k}) \right)c_{\vec{k} + \vec{Q}\uparrow}^+ c_{\vec{k}\downarrow} = 0 \]

An exact SU(2) symmetry

The three conserved quantities are one in plane uniform spin density and the amplitude and phase of a helical spin density wave.
The Non-Abelian Gauge Transformation

$H_{ReD}$ in the form of a background non-abelian gauge potential

$$H_{ReD} = \frac{k^2}{2m} + \frac{1}{2m} \left( k_+ - 2m \alpha \sigma_z \right)^2 + \text{const.}$$

- Field strength vanishes; eliminate the vector potential by non-abelian gauge transf

$$\Psi_\uparrow(x_+,x_-) \rightarrow \exp(i2m \alpha x_+) \Psi_\uparrow(x_+,x_-) \quad \Psi_\downarrow(x_+,x_-) \rightarrow \exp(-i2m \alpha x_+) \Psi_\downarrow(x_+,x_-)$$

$$H_{ReD} \rightarrow H = \frac{k^2}{2m}$$

$$S^-(\vec{x}) = \psi_\downarrow^+(\vec{x}) \psi_\uparrow^-(\vec{x}) \rightarrow \exp(-i4m \alpha x_+) S^-(\vec{x})$$

$$S^+(\vec{x}) = \psi_\uparrow^+(\vec{x}) \psi_\downarrow^-(\vec{x}) \rightarrow \exp(i4m \alpha x_+) S^+(\vec{x})$$

- Mathematically, the PSH is a direct manifestation of a non-abelian flux in the ground state of the models.

The Boltzmann Transport Equations

For arbitrary $\alpha, \beta$ spin-charge transport equation is obtained for diffusive regime

For Dresselhaus $= 0$, the equations reduce to Burkov, Nunez and MacDonald, PRB 70, 155308 (2004); Mishchenko, Shytov, Halperin, PRL 93, 226602 (2004)

\[ \partial_t n = D \partial_i^2 n + B_1 \partial_{x_+} S_{x_-} - B_2 \partial_{x_-} S_{x_+} \]

\[ \partial_t S_{x_-} = D \partial_i^2 S_{x_-} + B_1 \partial_{x_+} n - C_2 \partial_{x_-} S_z - T_2 S_{x_-} \]

\[ \partial_t S_{x_+} = D \partial_i^2 S_{x_+} - B_2 \partial_{x_-} n - C_1 \partial_{x_+} S_z - T_1 S_{x_+} \]

\[ \partial_t S_z = D \partial_i^2 S_z + C_2 \partial_{x_-} S_{x_-} + C_1 \partial_{x_+} S_{x_+} - \left( T_1 + T_2 \right) S_z \]

\[ B_1 = 2(\alpha - \beta)^2 (\alpha + \beta) k_F^2 \tau^2, \]

\[ B_2 = 2(\alpha + \beta)^2 (\alpha - \beta) k_F^2 \tau^2, \]

\[ C_1 = 2(\alpha + \beta) k_F^2 \tau / m, \]

\[ C_2 = 2(\alpha - \beta) k_F^2 \tau / m, \]

\[ T_1 = 2(\alpha + \beta)^2 k_F^2 \tau, \]

\[ T_2 = 2(\alpha - \beta)^2 k_F^2 \tau \]
The Boltzmann Transport Equations

Along special directions the four equations decoupled to two by two blocks

Propagation on [1\bar{1}0]

\[ q_{\bar{n}} = q, q_{n} = 0 \]
\[ n \leftrightarrow S_{x_z}, S_{z} \leftrightarrow S_{x_{\bar{n}}} \]
\[ i\omega_{1,2} = -Dq^2 - \frac{1}{2} \left( 2T_2 + T_1 \pm \sqrt{T_1^2 + 4q^2C_2^2} \right) \]
At \( \alpha=\beta \)
\[ i\omega_1 = -Dq^2 - T_1, i\omega_2 = -Dq^2 \]

The behavior of \( S_{z} \) is diffusive and exponentially decaying; this is the passive direction

\[ C_1 = 2(\alpha + \beta)k_F^2\tau/m, \quad T_1 = 2(\alpha + \beta)^2k_F^2\tau, \]
\[ C_2 = 2(\alpha - \beta)k_F^2\tau/m, \quad T_2 = 2(\alpha - \beta)^2k_F^2\tau \]

Propagation on [110]

\[ q_{\bar{n}} = q, q_{n} = 0 \]
\[ n \leftrightarrow S_{x_z}, S_{z} \leftrightarrow S_{x_{\bar{n}}} \]
\[ i\omega_{1,2} = -Dq^2 - \frac{1}{2} \left( 2T_1 + T_2 \pm \sqrt{T_2^2 + 4q^2C_1^2} \right) \]
At \( \alpha=\beta \)
\[ i\omega_1 = -Dq^2 + T_1 \mp C_1q \]

At the shifting wave-vector \( Q \)
\[ i\omega_2(q = 4mA = Q) = 0 \]

An infinite spin life-time of the Persistent Spin Helix; this is the active direction
Prediction of non-diffusive dynamics

A spin grating is a linear combination of the two helix modes.
This the data of our first observation of Shoucheng and Andrei prediction

\[ q = 0.6 \times 10^4 \text{ cm}^{-1} \]
Tuning for Rashba = Linear Dresselhaus

Electric field is generated by asymmetric doping

Tune Dresselhaus term by changing well width
Emergence of the persistent spin helix in semiconductor quantum wells

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New samples for current driven spin helices

double EBASE processing at Sandia (Luyi Yang)

Spin helix is translationally invariant

Helix energy is independent of its phase

Can we measure drift of spin helix induced by current?

2DEG

Ohmic contacts
Doppler velocimetry of spin helix

Measure Doppler shift of diffracted probe

$$\frac{\Delta \omega}{\omega} \approx 10^{-7}$$

Corresponds to ~ 1nm spatial resolution
Directly observe phase winding of drifting spin helix

Turning up the E field

$\varphi_0 = 0$: in phase

$= \frac{\pi}{2}$: out of phase

$E = 52$ V/cm
Directly observe phase winding of drifting spin helix

Turning up the E field

$\varphi_0 = 0$: in phase

$= \frac{\pi}{2}$: out of phase

PRL 109, 246603 (2012)
Nature Physics 8, 153 (2012)
What about $\alpha \neq \beta$?

Spin propagation Rashba only


Precession angle *is* path dependent...

...leading to weaker, *but nonzero*, spin/space correlations.
Technical innovations

Phase mask array for rapid variation of $q$

Phase-modulated heterodyne detection of diffracted wave

Spin-momentum coupling from crystal field

Dresselhaus coupling in 2 and 3 dimensions

3D: Spin-Velocity interaction is cubic in velocity.

Doesn’t look like effective B field

\[ \mathcal{H}_{\text{Dresselhaus}} = \gamma (\sigma_x k_x (k_y^2 - k_z^2) + \text{c.p.}) \]

2D:
Linear term proportional to \(1/d^2\)

Cubic term proportional to \(k_F^2\)

\[ \mathcal{H}_{001} = \gamma (k_z^2) (\sigma_y k_y - \sigma_x k_x) + \gamma (\sigma_x k_x k_y^2 - \sigma_y k_y k_x^2) \]
Symmetric doping
Asymmetry introduces Rashba “spin-orbit” coupling