Theory of the supercyclotron resonance and Hall response in anomalous 2d metals

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“Do not grow old, no matter how long you live. Never cease to stand like curious children before the Great Mystery into which we were born.”

— Albert Einstein
Failed superconductors?

Weakly disordered amorphous InO$_x$: ‘anomalous’ metallic state at low temperatures in fields $H \gtrsim 2T$. (also in other superconducting films [Kapitulnik-Kivelson-Spivak])

[Graph showing temperature dependence of resistivity for InO$_x$ at various magnetic fields]
Slow phase relaxation

The anomalous metal displays a Lorentzian conductance peak

Peak defines a lifetime that is $10^5$ times greater than the quasiparticle lifetime

Simplest interpretation: long-lived collective ‘superfluid’ phase mode
i.e. indeed a failed superconductor
Vortices relax phase

\[ \sigma = \frac{f_s}{-i\omega + \Omega} \]

- superfluid stiffness
- phase relaxation rate = vortex conductivity

\[ j^x_v = \Omega \partial_y \phi \]

Understood for a long time [Bardeen-Stephen '65]:

\[ \Omega = x \frac{2f_s}{\sigma_n} \]

- fraction of area in mobile vortex cores
- conductivity in core

(small \(\Rightarrow\) slow phase relaxation)
Vortices relax phase

w/ Davison, Delacrétaz, Goutéraux

Slow phase relaxation
→ Kubo formula for $\Omega$ from hydrodynamic arguments
→ correlation functions of the operator:

$$\dot{J}_\phi = \frac{2}{\chi} \int_{\text{cores}} \nabla n \ d^2x$$

For large cores, the correlation functions of $n$ in the core takes a universal form → rederived Bardeen-Stephen
Hall response

Hall resistivity 3 orders of magnitude below conventional high T phase

Scaling $\rho_{xy} \sim \rho_{xx}^2$ in anomalous metal until signal below experimental sensitivity.

Weak H dependence on top of scaling, possibly $\sigma_{xy}$ vanishes below $H_{M2}$.

[Breznay-Kapitulnik '17]
**Flux flow Hall response**

*Extensive* theoretical effort to describe the Hall effect in flux flow regimes (motivated by e.g. cuprates):


Many different physical effects considered, capturing aspects of observed data. However, *domain of validity of computations generally unclear*, leading to contradictory results.

**Our strategy:** *systematic theory* based on diluteness of vortices (small $x$), and corresponding *slow phase relaxation*. 
Flux flow Hall response

w/ Luca Delacretaz

Extended (and clarified) our previous discussion.

‘Generalized Ohm law’:

\[
\begin{pmatrix}
    j^i_o \\
    j^i_v
\end{pmatrix} =
\begin{pmatrix}
    \sigma^{ij}_o & \alpha^{ij}_v \\
    \alpha^{ij}_v & \Omega^{ij} / f_s
\end{pmatrix}
\begin{pmatrix}
    E^j \\
    f_s \epsilon^{jk} \partial_k \phi
\end{pmatrix}
\]

The transport coefficients are given by Kubo formulae.
Summary of results I

When vortices are (i) dilute and (ii) large in units of normal state mean free path, then:

$$\rho_{yx} = \sigma_n^H \rho_{xx}^2 + \frac{n_v^{\text{eff}}}{n_s} \frac{\hbar}{e_s^2}$$

(A)

$n_v^{\text{eff}}$ is a thermodynamic susceptibility that determines how efficiently vortices are dragged by the superfluid.

With Galilean invariance: $n_v^{\text{eff}} = n_v \sim x \sim \rho_{xx}$
Summary of results II

With disorder, $n^\text{eff}_v$ can be small. Our result therefore allows $\rho_{yx} \sim \rho_{xx}^\beta$, $1 \leq \beta \leq 2$, observed across materials.

YBa$_2$Cu$_3$O$_7$  [Luo et al. PRL ’92]

Irradiated HgBa$_2$CaCu$_2$O$_{6+\delta}$  [Kang et al. PRB ’99]
(B) A ‘super-cyclotron’ resonance in the optical response:

\[ \omega = \Omega_H \equiv \frac{n_{v}^{\text{eff}}}{m_\ast}, \quad \Delta \omega = \Omega \]

This is analogous to the cyclotron resonance, but the superfluid velocity plays the role of the normal velocity.

(C) Optical data [Wang-Tamir-Shahar-Armitage ’17]: \( \Omega_H \lesssim 10^{-5} \Omega \)

(incompatible with conventional metal, which would have had a cyclotron rather than Drude response)

\[ \Rightarrow \frac{n_{v}^{\text{eff}}}{n_v} \lesssim 10^{-5} \]
Summary of results IV

This small value of $n_{v}^\text{eff}$ then implies that $\rho_{yx} \sim \rho_{xx}^{2}$, as is observed [Breznay-Kapitulnik ’17].

(quantitatively, that data requires $n_{v}^\text{eff} \lesssim 10^{-4}n_{v}$)

Prediction: Supercyclotron resonance should be seen in materials closer to free flux flow, with $\rho_{yx} \sim \rho_{xx}$. 
Evidence that transport in the anomalous metal in InO$_x$ is due to (non-Galilean invariant) vortex flux flow:

1. ‘Drude’ peak too narrow for quasiparticles.
2. Peak at zero frequency rather than cyclotron-like.
3. Scaling of Hall resistivity: $\rho_{yx} \sim \rho_{xx}^2$

Developed systematic, controlled theory of flux flow transport in magnetic fields, based on slow phase relaxation, connecting dc and finite frequency transport.
Happy Birthday Aharon!